

Today's topic is Arithmetic Progression (AP). An AP is a sequence of numbers such that the difference between the consecutive terms is constant.

For example:

2, 4, 6, 8...

-1, 10, 21, 32...

4, 3, 2, 1, 0, -1, -2, -3

$-1/2, -3/2, -5/2, -7/2, \dots$

and so on.

Note that the numbers could be increasing or decreasing. As long as the difference between the consecutive terms is constant, it is an AP. The general form of an AP is given by

$a, a+d, a+2d, a+3d, \dots$

I hope you see how we get it. 'a' is the first term and 'd' is the common difference. The nth term has the form ' $a + (n-1)d$ '. The first term is given by ' $a + (1-1)d = 'a'$ '

The second term is given by ' $a + (2-1)d = 'a + d'$ ' etc

The sum of n terms will be found by doing the following:

$a + (a+d) + (a+2d) + \dots + (a+(n-1)d) = na + (d + 2d + 3d + \dots + (n-1)d)$ (summing the n first terms together and clubbing the common differences together in a bracket)

$= na + d(1 + 2 + 3 + \dots (n-1))$ (taking d common)

$= na + d(n-1)n/2$ (this is because the sum of n consecutive integers starting from 1 is given by $n(n+1)/2$. We will work on this concept in detail next week.)

$= n/2 (2a + (n-1)d)$

If you notice carefully, $2a + (n-1)d$ can be re-written as: $[a] + [a + (n-1)d]$ i.e. the sum of first and last terms. So basically the sum of n terms of the AP is $[n * (\text{First term} + \text{Last term})/2]$ which is the same as $[\text{number of terms} * \text{Average of the first and the last terms}]$.

Questions on APs are very simple. Sometimes, people just don't realize that the question is based on an AP. The questions in GMAT may not give you that the sequence is an AP but it is not tough to figure out. Let us look at an example now.

Question 1: The nth term, $t(n)$, of a certain sequence is defined as $t(n) = t(n-1) + 4$. If $t(1) = -11$, then $t(82) =$

- (A) 313
- (B) 317
- (C) 320
- (D) 321

(E) 340

Solution:

The given relation says that every n th term is 4 more than the previous term. So basically, it tells us that the sequence is an AP. Whew! (An AP is very easy to work with)

What is the n th term of an AP? It's $[a + (n-1)d]$

What is the 82nd term of this AP? It's $[-11 + 81 \cdot 4] = 313$

Answer (A)

Question 2: If S is the infinite sequence such that $t(1) = 4$, $t(2) = 10$, ..., $t(n) = t(n-1) + 6, \dots$, what is the sum of all the terms from $t(10)$ to $t(18)$?

(A) 671

(B) 711

(C) 738

(D) 826

(E) 991

Solution: Again, since the n th term is 6 more than the previous term, it is an AP.

We need to find the following sum: $t(10) + t(11) + t(12) + \dots + t(18)$

But we only know how to find the sum starting from $t(1)$. Let's manipulate what we have to find a little to make it similar to what we know.

$t(10) + t(11) + t(12) + \dots + t(18) = \text{Sum of first 18 terms} - \text{Sum of first 9 terms}$

Sum of first 18 terms = $(18/2)(2 \cdot 4 + 17 \cdot 6)$

Sum of first 9 terms = $(9/2)(2 \cdot 4 + 8 \cdot 6)$

$t(10) + t(11) + t(12) + \dots + t(18) = (18/2)(2 \cdot 4 + 17 \cdot 6) - (9/2)(2 \cdot 4 + 8 \cdot 6)$

$= 990 - 252 = 738$

Answer (C)

As you see, AP questions are easy to work with. Next week, we will discuss some properties of a specific type of AP i.e. consecutive integers. Till then, keep practicing!